Errata and updates for thesis

Last updated January 24, 2009.

Mathematical slips

- pages 12 and 14: there are slight inaccuracies in the hypotheses of Proposition 1.4.7 and Corollary 1.5.3; for the claimed isomorphism ${}_A\operatorname{Hom}(A_{\mathsf{un}}\widehat{\otimes}E, X)\cong\mathcal{L}(E,X)$ is in general only valid when X is A_{un}-unit-linked – as indeed it is in all the intended applications. Conceptually, one could fix this gap either by working throughout with the forced unitization $A^{\#}$, or by restricting attention throughout to modules which are A_{un}-unit-linked. Thus:
  - For Proposition 1.4.7: replace A_{un} by $A^{\#}$ throughout the statement of the proposition. Then add a remark to say that if M is unit-linked A_{un}-module, the proposition holds with $A^{\#}$ replaced by A_{un}. (This latter version is needed elsewhere in the thesis.)
  - For Corollary 1.5.3: the statement is correct, but in the proof one now uses $\mathcal{C}_*(A, A^{\#})$ and the corrected version of Proposition 1.4.7.

- Page 26, penultimate paragraph: while the result quoted from B.E. Johnson's 1972 monograph is correct, we actually need the 'opposite version' with left and right interchanged. The point is that Johnson prefers to reduce bounded cohomology to the case where the coefficient module has trivial *right* action; but for the purposes of §2.2.2 of the thesis, we instead want coefficients with trivial *left* action. It follows that on lines 3-4 from the end, we instead want $\widetilde{I_0(G)}$ to have augmentation action on the *right* and conjugation action on the *left*. This can be achieved by a left-right switch in Johnson's original formulas, and so the proof of Theorem 2.2.4 remains valid.

- Page 42: the last sentence of Theorem 3.2.2 is not quite right as it stands, since the functor $\_\_\widehat{\otimes}\mathsf{id}$ does not necessarily send linear maps with closed range to linear maps with closed range. The correct statement should be:

  > "If τ_A and τ_B have closed range, *and if the underlying Banach spaces of A and B are $\mathcal{L}_1$-spaces*, then τ_C has closed range", etc.

  The statement of Corollary 3.2.3 needs to be adjusted accordingly, but otherwise the proof of Proposition 4.1.3 (p. 55) goes through as before.

- Page 107, 1st para of §5.8: the reference to Gerstenhaber and Schack's paper (*Trans. AMS*, 1988) is somewhat misleading, as their construction is not really related to that in §5. For instance: if we have a presheaf over a semilattice L where each stalk is the trivial algebra $\mathbb{C}$, our construction is just the convolution algebra of L, whereas Gerstenhaber and Schack's seems to give the *incidence algebra* of L.

- Page 139, final para: the last paragraph is wrong – the proof of Sheinberg's theorem uses the hypothesis that $\mathcal{H}^1(A, \mathcal{L}(K, L)) = 0$ for all Hilbert modules K and L (not $\mathcal{H}^2$ as claimed in the text). It follows that there is no obvious reason to relate Sheinberg's theorem to the question of whether a proper uniform algebra can be smooth.

Clarifications, updated references, etc.

- Page 22, final paragraph: the fact that biflatness implies simplicial triviality can be found as Proposition 2.8.62 of

  DALES, H. G. *Banach algebras and automatic continuity*, LMS Monographs (New Ser.) no. 24, OUP 2000.

  where it is proved in essentially the same way as in Appendix B of the thesis.

- Page 31, remark after Corollary 2.2.10. There is an easier argument to show that $I_0(F_2)$ is not biflat: by Corollary 4.10(i) of

  SELIVANOV, YU. V. Cohomological characterizations of biprojective and biflat Banach algebras. *Monatsh. Math.*, 128(1):35–60, 1999.

  if A is a biflat Banach algebra then $\mathcal{H}^2(A, \mathbb{C}_{\mathsf{ann}}) = 0$ where $\mathbb{C}_{\mathsf{ann}}$ denotes the one-dimensional annihilator module. But a standard argument involving unitizations shows that $\mathcal{H}^2(I_0(F_2), \mathbb{C}_{\mathsf{ann}}) \cong \mathcal{H}^2(\ell^1(F_2), \mathbb{C}_1)$, where the action of $\ell^1(F_2)$ on $\mathbb{C}_1$ is given by augmentation; and this latter cohomology group is known to be non-zero by the computations in Section 2 of B. E. Johnson's 1972 monograph.

- Page 60, lines 6–7 from end: we implicitly take it as read that if N is B-projective and P is C-projective, then $N\widehat{\otimes}P$ is $B\widehat{\otimes}C$-projective. This result is surely folklore but I haven't tracked down a reference for it. (One way to prove it is as follows: since N and P are projective, they are module-retracts of $B^\#\widehat{\otimes}N$ and $C^\#\widehat{\otimes}P$ respectively: now consider the composite map

  $$B^\#\widehat{\otimes}N\widehat{\otimes}C^\#\widehat{\otimes}P \to N\widehat{\otimes}C^\#\widehat{\otimes}P \to N\widehat{\otimes}P$$

  and construct a right-inverse module map to this composite, in the obvious way.)

- Page 64: Corollary 4.2.7 can be strengthened slightly and given a shorter proof. Namely, the following is true:

  If L is a right $\ell^1(\mathbb{Z})$-module, and we regard L as an $\ell^1(\mathbb{Z}_+)$-module via the inclusion homomorphism of $\ell^1(\mathbb{Z}_+)$into $\ell^1(\mathbb{Z})$, then L is $\ell^1(\mathbb{Z}_+)$-flat.

  Proof. Since $\ell^1(\mathbb{Z})$ is amenable every module over it is automatically flat (see Theorem VII.2.29 of Helemskii's book); and by Lemma 4.2.6 of the thesis, the inclusion homomorphism makes $\ell^1(\mathbb{Z})$ a flat module over $\ell^1(\mathbb{Z}_+)$. The result now follows by Proposition 4.18 of White's paper (*Proc. LMS*, 1996).

- Page 118: In Corollary 6.3.1, the hypothesis that A be amenable can be relaxed to biflatness: this follows from Theorem 4.13 of the aforementioned Selivanov paper.

Typographical errors

- Page 2, line 2 from end: it should be $(V, \|\_\_\|)$ rather than $(V, \_\_)$
- Page 3, line 2: it should be the 'indiscrete' topology rather than the 'discrete' one (that is, the coarsest possible topology).
- Page 4, line 13: it should be 'taken to be' rather than 'take to be'.
- Page 10, penultimate line: its should be $\pi : K\widehat{\otimes}K \to K$.
- Page 19, line 15: it should be 'for all $n \geq k$' rather than 'for all $n > k$'.
- Page 26, line 6 from end: it should be 'Proof of Theorem 2.2.4, assuming Corollary 2.2.6'.
- Page 27, line 10: 'succession' not 'succesion'.
- Page 27, line 3 from end: Equation (2.3) has been mangled, and should read

$$ {}_B\operatorname{Hom}(\_\_, \mathbb{C}) \underset{1}{\cong} {}_A\operatorname{Hom}\left(A \underset{B}{\widehat{\otimes}} \_\_, \mathbb{C} \right) $$

- Page 29, line 10: the comma between $\operatorname{Ext}^n_{\ell^1(G)}$ and $(\ell^1(S), \mathbb{C})$ should be omitted.
- Page 29, line 9 from end: delete the words 'of $\ell^1(G)$-modules'.
- Page 31, line 8: it should be $\overset{(\infty)}{\bigoplus}_{x\in\mathbb{I}}$ rather than $\overset{(1)}{\bigoplus}_{x\in\mathbb{I}}$.
- Page 33, line 11 from end: it should be $\operatorname{Ext}^{p+q}_R(B, {}_\varphi M)$.
- Page 39, final paragraph: it should be (B, X) twice, rather than (A, X).
- Page 40, line 7 from end: it should be σ_A (twice) rather than σ.
- Page 44, lines 5 and 6: the formulas on these lines have been mangled slightly. The correct replacement text is

$$ = \begin{pmatrix} \sigma_A(x_1 \otimes ax_2) \otimes y_1by_2 \\ x_1ax_2 \otimes \sigma_B(y_1 \otimes by_2) \end{pmatrix} - \begin{pmatrix} \sigma_A(x_1x_2 \otimes a) \otimes y_1y_2b \\ x_1ax_2 \otimes \sigma_B(y_1y_2 \otimes b) \end{pmatrix} - \begin{pmatrix} \sigma_A(ax_1 \otimes x_2) \otimes by_1y_2 \\ ax_1x_2 \otimes \sigma_B(by_1 \otimes y_2) \end{pmatrix} $$
$$ = \begin{pmatrix} (x_1 \otimes ax_2 - x_1x_2 \otimes a - ax_1 \otimes x_2 + ax_1x_2) \otimes by_1y_2 \\ ax_1x_2 \otimes (y_1 \otimes yb_2 - y_1y_2 \otimes b - by_1 \otimes y_2 + by_1y_2) \end{pmatrix} $$

- Page 45, 2nd diagram: $\widetilde{\mathsf{Ex}}$ should be $\widetilde{\mathsf{Ass}}$.
- Page 47, line 13: it should be $A^{\widehat{\otimes} 4}$ not $A^{\otimes 4}$.
- Page 56, line 8: it should be $q\rho_0 = \mathsf{id}$ rather than $q\rho = \mathsf{id}$.
- Page 86, line 11 from end: 'homomorphisms' not 'homomorhpisms'.
- Page 91, line 3 from end: it should be $e(0), \ldots, e(n) \in H$, not $e(0), \ldots, e(n) \in L$; and $a_j \in B_{e(j)}$, not $a_j \in A_{e(j)}$.

- Page 92, line 2 from end: ‘semigroup’ not ‘semigoup’.
- Page 95, line 7 from end: it should be $q_n : \mathcal{C}_n(\mathcal{B}, X) \to \mathcal{C}_n^F(\mathcal{B}, X)$.
- Page 99, line 2: it should be $b \in A_f$ rather than $b \in B_f$.
- Page 101: In Equation (5.8) b should be a throughout (to be consistent with the next page).
- Page 102, line 3: missing right bracket from end of equation.
- Page 102, line 10 from end: it should be $\coprod_{x \in L} A_x$.
- Page 104, lines 4–7 from end: it should be $\boldsymbol{\iota}_{\mathbf{f}}^{A\widehat{\mathbf{x}}}$ rather than $\boldsymbol{\iota}_{\mathbf{f}}^{B}$.
- Page 105, line 14: it should be $\mathcal{C}_*(\ell^1(G_e), \ell^1(G_e))$.
- Page 124, line 12: it should be “$\mathfrak{C}$ *is not* NC*-smooth*”.
- Page 128, line 9 from end: it should be $\mathcal{H}^2(\mathcal{V}, X) \neq 0$ rather than $\mathcal{H}^2(A, X) \neq 0$.
- Page 156, item 19: a more accurate transliteration is ‘A. Ya. Helemskii’ rather than ’A. Y. Helemskii’.