A decomposition theorem for quantum groups

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(joint work with Matthew Daws)

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Adam Skalski (Instytut Matematyczny PAN, Poland)
&
Stuart White (University of Glasgow, U.K.)

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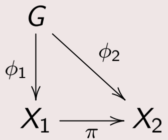

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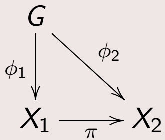

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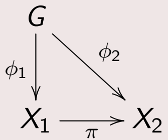

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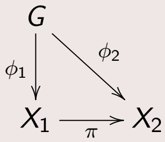

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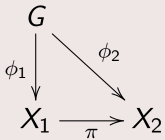

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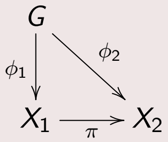

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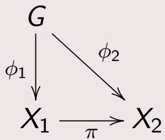

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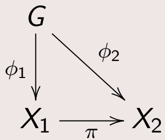

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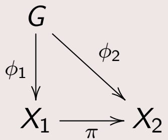

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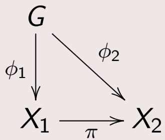

Initial object is Quantum Bohr compactification of G (P. Sołtan)

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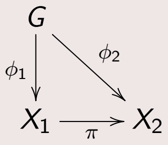

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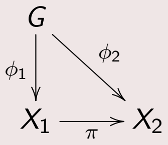

Do we still have an initial object?

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Definition (A) (Woronowicz; 1996)

A C* bi-algebra $(C_0(G), \Delta)$ such that

1. There exists a *manageable multiplicative unitary* $W \in B(H \otimes H)$ with $C_0(G) = [(\iota \otimes \omega)(W) : \ \omega \in B(H)_*]$
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Definition (B) (Kustermans & Vaes; 2000)

A C* algebra $C_0(G)$ and a non-degenrate *-homomorphism $\Delta : C_0(G) \longrightarrow M(C_0(G) \otimes C_0(G))$ satisfying:

1. $(\Delta \otimes \iota) \circ \Delta = (\iota \otimes \Delta) \circ \Delta$
2. $[\Delta(C_0(G))(1 \otimes C_0(G))] = C_0(G) = [\Delta(C_0(G))(C_0(G) \otimes 1)]$
3. There exists a faithful left-invariant approximate KMS weight ϕ on $(C_0(G), \Delta)$
4. There exists a right-invariant approximate KMS weight ψ on $(C_0(G), \Delta)$

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G $\longrightarrow$ reduced locally compact quantum group ; S $\rightarrow$ the antipode

$L^1_*(\widehat{G}) := \{\omega \in L^1(\widehat{G})\colon\ \overline{\omega} \circ S \subset f$ for some $f \in L^1(\widehat{G})\}$

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Definition (Kustermans; 2001)

A C* bialgebra $(C_0^u(G), \Delta_u)$ such that

1. $C_0^u(G)$ is the universal enveloping C* algebra of $L^1_*(\widehat{G})$
2. $\Delta_u : C_0^u(G) \longrightarrow M(C_0^u(G) \otimes C_0^u(G))$ is a non-degenerate *-homomorphism
3. $(\Delta_u \otimes \iota) \circ \Delta_u = (\iota \otimes \Delta_u) \circ \Delta_u$
4. $(\Lambda_G \otimes \Lambda_G) \circ \Delta_u = \Delta \circ \Lambda_G$

where $\Lambda_G : C_0^u(G) \longrightarrow C_0(G)$ is the reducing morphism and Δ is the coproduct of $C_0(G)$

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4. $(\Phi \otimes \iota)(V) = V_{13}V_{23}$ where Φ is "lifted"

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Theorem:

1. Let (A, Φ, V, H) be a compact *quantum* semi-topological (CH)-semigroup with A abelian.

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2. Let S be a compact semi-topological (CH)-semigroup acting on a Hilbert space H. For $s, t \in S$ let

   $$\Phi : C(S) \longrightarrow C(S)^{**}\overline{\otimes}C(S)^{**} : f \mapsto \Phi(f)(s,t) := f(s \cdot t).$$

   Then there exists a $V \in C(S)^{**}\overline{\otimes}B(H)$ with $\|V\| \leq 1$ such that
   $(C(S), \Phi, V, H)$ is a compact quantum semi-topological (CH)-semigroup.

Invariant means on (A, Φ, V, H)

Theorem (Existence of bi-invariant mean):

There exists a state $M : A \longrightarrow \mathbb{C}$

$$(\iota \otimes \widetilde{M})(\widetilde{\Phi}(a)) = (\widetilde{M} \otimes \iota)(\widetilde{\Phi}(a)) = M(a) \quad (a \in A)$$

where $\widetilde{M}$ and $\widetilde{\Phi}$ are the lifts of M and Φ to A^{**}.

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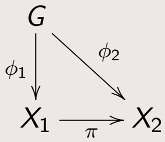

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$$S_1 := (A_1, \Phi_1, V_1, H_1) \; ; \; S_2 := (A_2, \Phi_2, V_2, H_2)$$

A morphism $\theta : S_1 \longrightarrow S_2$:

A unital *-homomorphism:

$$\theta : A_2 \longrightarrow A_1$$

satisfying

$$(\widetilde{\theta} \otimes \widetilde{\theta}) \circ \Phi_2 = \Phi_1 \circ \theta$$

where $\widetilde{\theta} : A_2^{**} \longrightarrow A_1^{**}$ is the lift of $\theta : A_2 \longrightarrow A_1$

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2. $(\widetilde{\Theta} \otimes \iota)(V)$ is a unitary representation of G

   The representation can be degenerate

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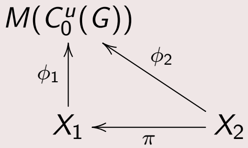

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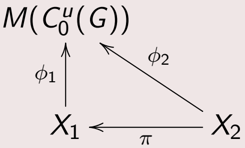

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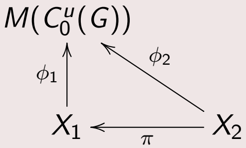

There exists initial object in this category (BD & Daws)

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3. If G is Kac-type then $E(G) = B(G)$

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   e.g. this is the case for G $:= \widehat{SU_q(2)}$
3. If G is Kac-type then $E(G) = B(G)$
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6. G is classical $\implies$ $E(G)$ is the continuous function algebra over the Eberlein compactification of G

Bohr compactification vs Eberlein compactification

Bohr compactification vs Eberlein compactification

de Leeuw & Glicksberg (1961); Spronk & Stokke (2012)

Let G be a locally compact group.

- Bohr(G) is the Bohr compactification of G
- E(G) is the continuous function algebra over the Eberlein compactification of G

Then

$$\mathbf{E(G) = C(Bohr(G))} \oplus \mathcal{I}_{\mathbf{0}} \quad \text{(as Banach spaces)}$$

where $\mathcal{I}_0$ is the kernel of the bi-invariant mean of $E(G)$.

Quantum Bohr compactification vs Quantum Eberlein compactification

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- The compact quantum group $E(G)/\mathcal{I}_0$ is quantum group isomorphic to $C(G^{\mathrm{SAP}})$

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Pre-decomposition theorem for quantum groups: (contd.)

If G is of Kac-type, then there exists a short exact sequence of C* algebras :

$$0 \longrightarrow \mathcal{I}_0 \stackrel{\iota}{\longrightarrow} E(G) \stackrel{\pi}{\longrightarrow} C(G^{\mathrm{SAP}}) \longrightarrow 0$$

where

- ι is the inclusion map
- π is the GNS representation of $E(G)$ associated with the bi-invariant mean

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$$\mathbf{E}(\mathbf{G}) = \mathbf{AP}(\mathbf{G}) \oplus \mathcal{I}_{\mathbf{0}}$$

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- If G is a classical locally compact group (de Leeuw & Glicksberg decomposition theorem)
- If G is the dual of a classical locally compact group H such that H_d (H with discrete topology) is amenable

THANK YOU FOR YOUR ATTENTION

Dziękuję za uwagę