QOP research network – 2014/15 meeting

Lancaster University, Wednesday 24th – Friday 26th September 2014

Programme

The QOP talks will be held in Lecture Theatre 2 and Lecture Theatre 5 of the Lancaster University Management School. The 50th Anniversary lectures will be held in the Frankland Lecture Theatre of the Faraday building.

QOP meeting, Wednesday 24th September (LUMS LT2)

1700 – 1750	Jonathan Partington	Zero-one laws for functional calculus on operator semigroups
	Dinner	

QOP meeting, Thursday 25th September (LUMS LT2)

0900 – 0950	Tomasz Kochanek	Towards a C^*-algebraic characterization of the Hyers–Ulam stability
1000 – 1050	Niels Laustsen	Ideals of operators on the Banach space of continuous functions on the first uncountable ordinal
	Refreshments	
1130 – 1220	Joel Feinstein	Convergence of the sequence of powers of an element of a Banach algebra
	Lunch	
1400 – 1450	Olga Maleva	Smallest universal differentiability sets in Banach spaces
1500 – 1550	Yemon Choi	Unitarizable representations and amenable operator algebras
	Refreshments	
1630 – 1720	Adam Skalski	Haagerup property for von Neumann algebras – old and new

QOP meeting, Friday 26th September (LUMS LT5)

1000 – 1050	Claus Köstler	Noncommutative de Finetti theorems
	Refreshments	
1130 – 1220	Biswarup Das	A quantum version of the de Leeuw–Glicksberg decomposition theorem
	Lunch	
1330 – 1420	Piotr Nowak	Bounded cohomology and amenable actions

End of the QOP meeting

50th Anniversary lectures, Friday 26th September (Frankland LT)

Welcome from Professor Amanda Chetwynd

1500 – 1600	E. Brian Davies	Non-self-adjoint spectral problems
	Refreshments	
1630 – 1730	W. Hugh Woodin	A half-century of independence

QOP lecturers

Yemon Choi	Lancaster University
Biswarup Das	Institute of Mathematics of the Polish Academy of Sciences
Joel Feinstein	University of Nottingham
Tomasz Kochanek	Institute of Mathematics of the Polish Academy of Sciences
Claus Köstler	National University of Ireland, Cork
Niels Laustsen	Lancaster University
Olga Maleva	University of Birmingham
Piotr Nowak	Institute of Mathematics of the Polish Academy of Sciences and University of Warsaw, visiting the University of Oxford
Jonathan Partington	University of Leeds
Adam Skalski	Institute of Mathematics of the Polish Academy of Sciences and University of Warsaw

50th Anniversary lecturers

Professor Brian Davies has a post-retirement, part-time position at King's College, London. He was the President of the London Mathematical Society during one of its most fraught recent periods, from 2007 to 2009; honours include a Fellowship of the Royal Society since 1995. He is the author of more than 200 mathematical papers, mostly devoted to analysis, heat equations and non-self-adjoint spectral theory, and he has written 7 books, including *Linear Operators and their Spectra*; his interests encompass the philosophy of mathematics - as seen in *Science in the Looking Glass* and *Why Beliefs Matter*. He is also well known in the quantum physics community for work that he did in the 1970s on quantum measurement and open quantum systems.

Professor W. H. Woodin holds a joint position in the Departments of Mathematics and of Philosophy at Harvard University; he was previously Professor at the University of California, Berkeley, from 1989 to 2014. In 1985 Professor Woodin was awarded the Presidential Young Investigator Award, and he received the Hausdorff Medal of the European Set Theory Society in 2013. He was a plenary lecturer at the International Congress of Mathematicians in 2010, and a section lecturer in 1986 and 2002. He featured in the BBC Horizon programme *To Infinity and Beyond* in 2010. Professor Woodin is one of the leaders in our era in the quest to understand the fundamental nature of sets and the real numbers, taking forward the journey of Gödel and Tarski into the far reaches of higher cardinals.

Abstracts – QOP meeting

Yemon Choi **Lancaster University**

Unitarizable representations and amenable operator algebras (Thursday 1500)

By an old result of Nagy, all bounded representations of $\mathbf{Z}$ on a Hilbert space H are unitarizable inside $B(H)$. One naturally wonders: is the same true for representations inside the Calkin algebra? What if we replace $\mathbf{Z}$ by some other discrete abelian group?

Farah and Ozawa recently showed that the answer to this last question is negative in general. I will explain how they combined this fact with an ingenious idea of Ozawa to produce the first known examples of amenable operator algebras that are not isomorphic to C^*-algebras. I will then present a sharper version, taken from joint work with Farah and Ozawa, where the group consists of commuting involutions, and where one obtains an amenable subalgebra of $\ell_\infty(\mathbf{N}, \mathbf{M}_2)$ that is not isomorphic to any C^*-algebra. This shows that the pathology does not depend on exotic noncommutative phenomena, but rather on the structure of $\beta\mathbf{N} \setminus \mathbf{N}$.

Biswarup Das **Institute of Mathematics of the Polish Academy of Sciences**

A quantum version of the de Leeuw–Glicksberg decomposition theorem (Friday 1130)

In 1961, de Leeuw and Glicksberg showed that the C^*-algebra of almost periodic functions on a topological group G is complemented (as a Banach space) inside the C^*-algebra of weakly almost periodic functions. Later in 2012, Stokke and Spronk adapted this argument and proved similar results: almost periodic functions are complemented inside the Eberlein compactification of G.

Using C^*-algebraic arguments, we will generalize this decomposition result in the context of Kac algebras. In particular, we will recover the classical decomposition theorem as a corollary. Our methods give a new proof of this decomposition result for classical groups. (Based on joint work with Matthew Daws.)

Joel Feinstein **University of Nottingham**

Convergence of the sequence of powers of an element of a Banach algebra (Thursday 1130)

This is joint work with David Moore (one of my current PhD students).

Recall that a bounded linear operator T on a Banach space is said to be *quasicompact* if the essential spectral radius of T is strictly less than 1.

Our discussion has its origins in the work of Kamowitz (beginning in the 1970s) on compact endomorphisms of commutative, semisimple Banach algebras. Since then, a number of authors have studied various classes of endomorphisms of Banach algebras. In particular, Feinstein and Kamowitz (2010) obtained what appear to be the definitive results concerning the sequence of powers of quasicompact endomorphisms of commutative, semiprime Banach algebras.

We discuss a new approach to these results based on the holomorphic functional calculus and spectral projections. This approach allows us to recover and extend the main result of Feinstein and Kamowitz concerning the sequence of powers of quasicompact endomorphisms of Banach algebras.

We are also able to prove results concerning the behaviour of the sequence of powers of an element of a Banach algebra. We give some sufficient conditions for the convergence of such a sequence of powers. We also take a closer look at the behaviour of the sequence of powers of a bounded operator on a Banach space.

Tomasz Kochanek **Institute of Mathematics of the Polish Academy of Sciences**

Towards a C^*-algebraic characterization of the Hyers–Ulam stability (Thursday 0900)

We say that a (discrete) group G has the *Hyers–Ulam property* provided that for every map $\phi : G \to \mathbb{R}$ satisfying

$$\sup\{|\phi(xy) - \phi(x) - \phi(y)| : x, y \in G\} < \infty$$

we have $\operatorname{dist}(\phi, \operatorname{Hom}(G, \mathbb{R})) < \infty$. We will discuss some results concerning the following question: given a group G, what kind of statements about its reduced (full) C^*-algebra translate exactly into the Hyers–Ulam property of G? One source of motivation is the recent work by N.P. Brown and E.P. Guentner [*New C^*-completions of discrete groups and related spaces*, Bull. London Math. Soc. 45 (2013), 1181–1193] which gives the very first C^*-algebraic characterization of the so-called Haagerup property, or a-T-menability, as well as a new C^*-algebraic characterization of amenability. (Recall that all amenable groups enjoy the Hyers–Ulam property.) The core of our investigation lies in the work of Ch. Bavard [*Longueur stable des commutateurs*, Enseign. Math. 37 (1991), 109–150] who used the theory of bounded cohomology to characterize the Hyers–Ulam property by vanishing of the stable commutator length on the commutator subgroup $[G, G]$ of G.

Claus Köstler **National University of Ireland, Cork**

Noncommutative de Finetti theorems (Friday 1000)

The classical de Finetti theorem characterizes an exchangeable sequence of random variables to be identically distributed and conditionally independent over its tail algebra. Here a sequence is exchangeable if its joint distribution is invariant under permutations of the random variables. Stoermer provided such a result for symmetric states on the infinite minimal tensor product of unital C^*-algebras and identifies symmetric states as convex combinations of infinite tensor product states. Recently Rolf Gohm and I have provided noncommutative de Finetti theorems in the broader framework of W^*-algebraic probability spaces, generalizing exchangeability to braidability or even further to spreadability. Moreover, in the context of W^*-algebraic probability spaces, Roland Speicher and I characterized freeness with amalgamation via quantum exchangeablity where the role of permutations in the classical setting is replaced by Wang's quantum permutations. A natural question is to ask to what extent these results transfer to the framework of C^*-algebraic probability spaces. My talk will introduce to the general background of noncommutative de Finetti theorems and report on recent progress made in the study of quantum symmetric states on the infinite universal free product of a unital C^*-algebra. My talk is based on joint work with Ken Dykema, Rolf Gohm, Roland Speicher and John Williams.

Niels Laustsen **Lancaster University**

Ideals of operators on the Banach space of continuous functions on the first uncountable ordinal

(Thursday 1000)

I shall report on joint work with Tomasz Kania (Lancaster) and Piotr Koszmider (IMPAN, Warsaw), in which we study the lattice of closed ideals of the Banach algebra $\mathscr{B}(C_0[0,\omega_1))$ of bounded operators acting on the Banach space $C_0[0,\omega_1)$ of scalar-valued, continuous functions which are defined on the locally compact ordinal interval $[0,\omega_1)$ and vanish eventually. (Here ω_1 denotes the first uncountable ordinal.) Our main theorem gives a number of equivalent conditions, each describing the unique maximal ideal $\mathscr{M}$ of $\mathscr{B}(C_0[0,\omega_1))$. Among the consequences of this result are that $\mathscr{M}$ has a bounded left approximate identity (this complements a 25-year old result of Loy and Willis stating that $\mathscr{M}$ has a bounded right approximate identity) and that $\mathscr{B}(C_0[0,\omega_1))$ has a unique second-largest ideal.

Olga Maleva **University of Birmingham**

Smallest universal differentiability sets in Banach spaces (Thursday 1400)

In a given space X, we are looking for as small as possible *universal differentiability sets* (UDS) S, defined by the requirement that every Lipschitz function on X has a point of differentiability in S. We show that all spaces X with separable dual contain infinite-dimensional fractal universal differentiability sets of Hausdorff dimension 1. This is the lowest possible as all projections of the set of differentiability points inside UDS have positive measure. We also show that the 1-dimensional Hausdorff measure of a UDS can never be sigma-finite and discuss further improvements in dimension when X is a finite-dimensional space.

Piotr Nowak **Institute of Mathematics of the Polish Academy of Sciences and University of Warsaw, visiting the University of Oxford**

Bounded cohomology and amenable actions (Friday 1330)

In 1972 B.E. Johnson proved that amenable groups are precisely those for which bounded cohomology with coefficients in any dual module vanishes. This result allowed him to define amenability of a Banach algebra in terms of vanishing of Hochschild cohomology. Topologically amenable group actions are generalizations of amenability that play an important role the context of the Baum–Connes conjecture. In this talk I will present a characterization of amenable actions in terms of vanishing of bounded cohomology. This characterization reduces to Johnson's theorem in the case of actions on a point, and in the case of amenability of an action on the Stone–Cech compactification of the group it gives a cohomological description of exact groups, i.e., groups whose reduced group C^* algebra is exact. This answers a question of N.-Higson and is based on results obtained jointly with Ron Douglas, and Jacek Brodzki, Graham Niblo and Nick Wright.

Jonathan Partington **University of Leeds**

Zero-one laws for functional calculus on operator semigroups (Wednesday 1700)

For a strongly continuous semigroup of operators defined on a Banach space, it has long been of interest to obtain qualitative information (such as boundedness of the generator) from estimates of operator-valued functions defined in terms of the semigroup.

We present some classical and recent results for semigroups defined on the half-line or a sector: these include norm estimates of functions defined in terms of Laplace transforms of measures.

The talk is based on joint work with Isabelle Chalendar (Lyon) and Jean Esterle (Bordeaux).

Adam Skalski **Institute of Mathematics of the Polish Academy of Sciences and University of Warsaw**

Haagerup property for von Neumann algebras – old and new (Thursday 1630)

The Haagerup approximation property for a von Neumann algebra with a fixed faithful tracial state was introduced over 30 years ago by M. Choda. The origins of this concept lie on one hand in the work of A. Connes and V. Jones on von Neumann algebraic property (T) and on the other in the Haagerup property for discrete groups. Recent study of the latter property in the world of quantum groups led naturally to the introduction of the Haagerup property for a von Neumann algebra with a fixed normal faithful semifinite weight. We will describe the latter, show that it does not depend on the choice of the weight and present some equivalent characterisations. Based on joint work with Martijn Caspers (and also Rui Okayasu and Reiji Tomatsu).

Abstracts – 50th Anniversary lectures

E. Brian Davies **King's College London**

Non-self-adjoint spectral problems (Friday 1500)

Non-self-adjoint spectral theory is not yet a coherent subject, in the sense that there is no analogue of the spectral theorem that can act as a basis for further research. The very high instability of eigenvalues under small perturbations often affects the analysis of particular models. The number of these understood is expanding rapidly and will continue to do so into the foreseeable future.

We describe joint work with Michael Levitin on the $N \to \infty$ asymptotic spectral behaviour of a particular family of large non-self-adjoint matrices $A_{c,N}$ associated with a self-adjoint linear pencil. Crucial insights were obtained by numerical experiments, even though the final analysis does not rely on numerics. The problem is a matrix analogue of an indefinite self-adjoint linear pencil that concerns a Dirac operator with an indefinite potential. In some sense it is the simplest matrix example of its type, but its behaviour is still far more complex than one might expect. The eigenvalues of the matrix $A_{c,N}$ converge to the real axis as $N \to \infty$, but the details of the convergence depend strongly on the choice of the real parameter c, in a way that presently defies understanding, even at a numerical level.

W. Hugh Woodin **Harvard University**

A half-century of independence (Friday 1630)

Just over 50 years ago, Paul Cohen announced the unsolvability of Cantor's Continuum Hypothesis on the basis of the basic principles of Set Theory, these are the ZFC axioms.

But does this mean that the question of the Continuum Hypothesis has no answer? Any "solution" must involve the adoption of new principles but which principles should one adopt? Alternatively, perhaps the correct assessment of Cohen's discovery is that the entire enterprise of the mathematical study of Infinity is ultimately doomed because the entire subject is simply a human fiction without any true foundation. In this case, any "solution" to the Continuum Hypothesis is just an arbitrary (human) choice.

Over the last few years a scenario has emerged by which with the addition of a single new principle not only can the problem of the Continuum Hypothesis be resolved, but so can all of the other problems which Cohen's method has been used to show are also unsolvable (and there have been many such problems). Moreover the extension of the basic (ZFC) principles by this new principle would be seen as a compelling option based on the fundamental intuitions on which the entire mathematical conception of Infinity is founded.

However, this scenario critically depends upon the outcome of a single conjecture. If this conjecture is false then the entire approach, which itself is the culmination of nearly 50 years of research, fails or at the very least has to be significantly revised.

Thus the mathematical study of Infinity has arguably reached a critical point and interesting times are ahead.