Representation Theory and Symplectic Singularities

4th-8th April 2016, Edinburgh

Titles and Abstracts

Monday 4th April

10-11am *Stephen Donkin*, (University of York)

Title: Injective Schur Modules

Abstract: This is joint work with H. Geranios. This is a topic in the polynomial representation theory of a general linear group GL(n) over an infinite field of positive characteristic. The irreducible polynomial GL(n)-modules are labelled by the partitions of length at most n. For each such partition one has the induced module (or Schur module). This has simple socle and so embeds in the corresponding injective indecomposable module in the polynomial category. We are interested in the situation in which this embedding is an isomorphism, i.e., when the Schur module is injective. We show that this property does not depend on the length of the partition and give a combinatorial description of the partitions with the injectivity property. The arguments use the infinitesimal subgroups of GL(n) and there are interesting connections with the representation theory of symmetric groups.

11.30am-12.30pm *Alexey Sevastyanov* (University of Aberdeen)

Title: An explicit description of Poisson q-W algebras and their quantization

Abstract: I shall give an explicit formula for the Poisson bracket for Poisson q-W algebras and discuss its quantization. In particular, it will be shown that the centre of the quantum group is isomorphic to the representation space for modified q-Toda Hamiltonians.

3-4pm *Donna Testerman* (École Polytechnique Fédérale de Lausanne)

Title: Multiplicity free actions of simple algebraic groups

Abstract: We discuss joint work with Martin Liebeck and Gary Seitz, in which we consider the following question:

Let M be a maximal closed reductive subgroup of a simple algebraic group G, defined over an algebraically closed field k of characteristic 0*. Determine all irreducible kG-modules V such that V is a multiplicity-free kM-module.*

We report on general techniques for approaching the problem and our results for the case where $G = \mathrm{SL}(W)$ and M is a simple group of type A_n acting irreducibly on W.

4.15-5.15pm *Simon Goodwin* (University of Birmingham)

Title: Minimal dimensional modules for reduced enveloping algebras for $\mathfrak{gl}_n$

Abstract: We discuss a classification of minimal dimensional modules for reduced enveloping for $\mathfrak{gl}_n$. As a consequence we observe that they are all induced for from 1-dimensional modules of parabolic subalgebras. This is joint work with Lewis Topley.

Tuesday 5th April

9-10am *Michel Brion* (Institut Fourier)

Title: Commutative algebraic groups up to isogeny

Abstract: The commutative algebraic groups over a field k form an abelian category C. When k is algebraically closed, the homological dimension of C is 1 in characteristic 0 (Serre) and 2 in positive characteristics (Oort). Over a perfect field, this homological dimension can be arbitrarily large (Milne). The talk will be devoted to the quotient of C by the Serre subcategory F of finite algebraic groups. In particular, we will see that the homological dimension of C/F is 1 for any field k.

10.15-11.15am *Oksana Yakimova* (Friedrich-Schiller-Universität Jena)

Title: Polynomiality condition for rings of symmetric invariants

Abstract: Let $\mathfrak{q}$ be a Lie algebra. We say that $\mathfrak{q}$ satisfies the polynomiality condition if the ring $S(\mathfrak{q})^{\mathfrak{q}}$ of symmetric $\mathfrak{q}$-invariants is a polynomial ring in r variables, where r is the index of $\mathfrak{q}$. For some classes of Lie algebras, e.g. centralisers of elements $x \in \mathfrak{g}$ ($\mathfrak{g}$ being reductive), one finds a dichotomy, either the condition holds, or it is not clear whether $S(\mathfrak{q})^{\mathfrak{q}}$ is finitely generated or not. We will discuss other advantages of polynomiality and then concentrate on semi-direct products $\mathfrak{g} \ltimes V$, where V is a $\mathfrak{g}$-module.

11.45am-12.45pm *Dmitriy Rumynin* (University of Warwick)

Title: Localisation for Kac-Moody Groups

Abstract: We will look at representation theory of a complete Kac-Moody group G over a finite field. G is a locally compact totally disconnected group. After defining the group we will look at localisation of its category of smooth representations.

3-4pm *Claudio Procesi*, (Sapienza - Università di Roma)

Title: New and old trends in PI theory

Abstract: I will review the main results of the theory of algebras with polynomial identities, give some ideas of the current state of the art and open problems.

4.15-5.15pm *Georgia Benkart*, (University of Wisconsin-Madison)

Title: McKay-Steinberg Dynamics

Abstract: This talk will focus on Steinberg's result on representation graphs of finite groups, which was inspired by the McKay Correspondence, and various applications of it to invariant theory, centralizer theory, and discrete dynamical systems.

Wednesday 6th April

9.00-10.00am *David Stewart* (University of Newcastle)

Title: Maximal subalgebras of exceptional Lie algebras

Abstract: The question of classifying maximal subalgebras of Lie algebras goes all the way back to Sophus Lie himself and has a long history from Dynkin onwards. We report on the latest developments in classifying the maximal subalgebras of Lie algebras of simple algebraic groups over algebraically closed fields of positive characteristic, a reasonable task thanks to the Premet-Strade classification of simple Lie modular algebras in characteristics at least 5. This is joint work with Sasha Premet.

10.15-11.15am *Ivan Losev* (Northeastern University)

Title: Deformations of symplectic singularities and the orbit method

Abstract: Symplectic singularities were introduced by Beauville in 2000. These are especially nice singular Poisson algebraic varieties that include symplectic quotient singularities and the normalizations of orbit closures in semisimple Lie algebras. Poisson deformations of conical symplectic singularities were studied by Namikawa who proved that they are classified by points of a vector space. Recently I have proved that quantizations of conical symplectic singularities are still classified by the points of the same vector spaces. I will explain these results and then apply them to establish a version of Kirillov's orbit method for semisimple Lie algebras.

11.45am-12.45pm *Tomoyuki Arakawa* (Research Institute for Mathematics Sciences, Kyoto University)

Title: Deligne exceptional series, Feigin-Frenkel conjecture and W-algebras

Abstract: The associated variety of a vertex algebra is an important invariant that may be regarded as a generalization of associated varieties of primitive ideals of enveloping algebras. Although it is merely a Poisson variety in general, physicists expect that there should be a large number of interesting vertex algebras whose associated varieties are symplectic (in fact, hyperKähler). In this talk I will talk about some examples of such vertex algebras in relation with Feigin-Frenkel conjecture on Kac-Wakimoto admissible representations and the Deligne exceptional series, and apply them to (affine) W-algebras.

This is based on a joint work with Anne Moreau.

Thursday 7th April

9.00-10.00am *Victor Ginzburg* (University of Chicago)

Title: Indecomposable objects and potentials over a finite field

Abstract: Given an additive Karoubian category such that the moduli space of its objects is a smooth Artin stack (and some additional conditions) we give a formula for an exponential sum over the set of absolutely indecomposable objects of the category in terms of the geometry of the cotangent bundle on the moduli stack of framed objects. Our formula, inspired by the work of Hausel, Letellier, and Rodriguez-Villegas, provides a new approach for counting absolutely indecomposable quiver representations, parabolic bundles on a projective curve, and irreducible l-adic local systems (via a result of Deligne). Our approach is based on the formalism of factorization sheaves.

10.15-11.15am *Vera Serganova* (University of California, Berkeley)

Title: New tensor categories related to orthogonal and symplectic groups and the strange supergroup $P(\infty)$

Abstract: We study a symmetric monoidal category of tensor representations of the ind group $O(\infty)$. This category is Koszul and its Koszul dual is the category of tensor representations of the strange supergroup $P(\infty)$. This can be used to compute Ext groups between simple objects in both categories.

   The above categories are missing the duality functor. It is possible to extend these categories to certain rigid tensor categories satisfying a nice universality property. In the case of $O(\infty)$ such extension depends on a parameter t and is closely related to the Deligne's category $\operatorname{Rep} O(t)$. When t is integer, this new category is a highest weight category. When t is integer, the action of translation functors in this category is related to the representation of $\mathfrak{gl}(\infty)$ in the Fock space.

11.45am-12.45pm *Serge Skryabin* (Kazan (Volga Region) Federal University)

Title: Variation of numeric invariants in a flat family of finite dimensional algebras

Abstract: In this talk I am going to discuss the behaviour of Cartan invariants and the dimensions of simple and projective modules for finite dimensional associative algebras constituting a flat parametric family. There are some applications to generically semisimple families.

3.00-4.00pm *Henning Haahr Andersen*

Title: Tilting modules for algebraic groups and applications

Abstract: Let G be a reductive algebraic group over a field of prime characteristic. The endomorphism ring of a finite dimensional tilting module for G is a cellular algebra. This allows us to prove structural properties of several classes of finite dimensional algebras. For instance we obtain the conditions which ensure that the Brauer algebra (over any field) is semisimple.

4.15-5.15pm *Sasha Kleshchev* (University of Oregon)

Title: Khovanov-Lauda-Rouquier algebras and blocks of Hecke algebras

Abstract: We begin with a discussion of standard module theory for KLR algebras of finite and affine types, its connections with PBW bases in quantum groups, and affine highest weight categories. Then we describe some applications to blocks of symmetric groups and Hecke algebras.

Friday 8th April

9.00-10.00am *Eric Friedlander* (University of Southern California)

Title: Support varieties for linear algebraic groups

Abstract: We present the beginnings of a theory of support varieties for certain linear algebraic groups over an algebraically closed field of positive characteristic. For such an algebraic group G and a rational G-module M, we consider the ind-variety of 1-parameter subgroups $V(G)$ and the subvariety $V(G)_M \subset V(G)$. This theory satisfies some of the standard properties enjoyed by support varieties for finite groups (and, more generally, for finite group schemes). We introduce two interesting classes of rational G-modules, those that are mock injective and those that are mock trivial. We also construct a companion theory $V^{coh}(G)_M \subset V^{coh}(G)$ of cohomological support varieties based on the rational cohomology of G. This theory is closely related to the G-coinvariants of $V(G)_M \subset V(G)$. Examples of $G = \mathbb{G}_a$ and $G = U_3$ will be considered.

10.15am-11.15 *Rolf Farnsteiner* (Christian-Albrechts-Universität zu Kiel)

Title: Indecomposable modules of domestic group schemes

Abstract: Let k be an algebraically closed field of characteristic $p \geq 3$. In 1991, A. Premet determined the Green ring of the restricted enveloping algebra $U_0(\mathfrak{sl}(2))$ and provided an explicit description of the indecomposable $U_0(\mathfrak{sl}(2))$-modules. Earlier work by Drozd, Fischer and Rudakov had essentially shown that the non-simple blocks of $U_0(\mathfrak{sl}(2))$ are Morita equivalent to the trivial extension $k[\bullet \rightrightarrows \bullet] \ltimes k[\bullet \rightrightarrows \bullet]^*$ of the path algebra of the 2-Kronecker quiver. This implies in particular that $U_0(\mathfrak{sl}(2))$ is an algebra of domestic representation type.

The tame-wild dichotomy suggests that a classification of indecomposable modules of algebras of tame representation type should be feasible. However, this problem remains open for seemingly harmless examples such as the group algebra of the quaternion group of order 8 over a field of characteristic 2. Domestic algebras, introduced by Ringel, are tame algebras whose number of one parameter-families of indecomposables of any given dimension is uniformly bounded. The purpose of this talk is to indicate how Premet's classification can be extended to finite group schemes of domestic representation type. The combinatorial data of the stable Auslander-Reiten quiver of such a group scheme are related to McKay quivers and the ramification indices associated to morphisms of support varieties.

11.45am-12.45pm *Helmut Strade* (Universität Hamburg)

Title: Alexander Premet and the Classification of simple modular Lie algebras

Abstract: In this talk we give a historical overview of the theory of Lie algebras over fields of positive characteristic. The main issue will be the classification of the simple Lie algebras over algebraically closed fields of characteristic > 3 and the contributions of A. Premet to this.