The Statistical Evaluation of Controlled Substance Traces on Banknotes

David Lucy, Department of Mathematics & Statistics, Lancaster University, LA1 4YW.

Introduction

Banknotes which are contaminated with controlled substances are used by investigators as a source of evidence in cases where the supply of controlled substances may be suspected, but no illicit substances found directly in the possession of, or in association with, any given suspect.

The statistical problem for forensic scientists is one of the evaluation of a sequence of concentrations for controlled substances within a suitable propositional framework suggested by the offence.

Likelihood ratio

Denote x as the concentration of controlled substance upon a banknote.

1. Let X_1 be a sample of transformed measurements from a sample of raw observations Y_1, from a source known to be associated with the supply of some specific controlled substance.
2. Let X_2 be a sample of transformed measurements from a sample of raw observations Y_2 from banknotes in general circulation.
3. Let X_3 a sample of transformed measurements from a sample of raw observations Y_3 from banknotes seized from a suspect.

$\hat{f}(X_3)$ can then be used as a weighting function to calculate a likelihood ratio:

$$\mathrm{LR} = \frac{Pr(X_3|H_p)}{Pr(X_3|H_d)} = \frac{\int \hat{f}(X_1)\hat{f}(X_3)\,dx}{\int \hat{f}(X_2)\hat{f}(X_3)\,dx}$$

For propositions:

1. $H_p \equiv$ the suspect banknotes came from a source associated with the supply of a specific controlled substance(s).
2. $H_d \equiv$ the suspect banknotes came from banknotes not associated with the supply of a specific controlled substance(s) (ie: from general background).

In practice the distribution of concentrations of any controlled substance upon any specific group of banknotes may be modelled by some uni-modal distribution, however, models from different families of models may be required for the different sources of banknotes. Therefore some non-parametric models, such as kernel density estimates, may be appropriate for X_1,X_2 and X_3.

Autocorrelation

Banknotes are stored in intimate proximity to one another, and consequently there is a tendency for controlled substance contamination to be transferred from one note to another. This effect is particularly pronounced for notes which are close in the sequence. The partial autocorrelations for separations 1 to 6 are given below for two samples of banknotes from Switzerland:

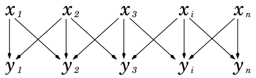
	separation (notes)					
source	1	2	3	4	5	6
Y_1	0.46	0.18	0.01	0.03	0.03	0.06
Y_2	0.38	0.18	0.12	0.09	0.07	0.07

Because the observed measurements of controlled substance are not the same as the initial concentrations, and different bundles of notes will experience different histories, even the relative concentrations between bundles will not be directly comparable.

Consider a sequence of n values $\mathbf{X} = \{x_1, x_2, \ldots, x_i, \ldots, x_n\}$ which become transformed through some function to a sequence of n observed concentration values, $\mathbf{Y} = \{y_1, y_2, \ldots, y_i, \ldots, y_n\}$.

Furthermore assume that there is no net loss of quantity, so that $\Sigma(\mathbf{Y}) = \Sigma(\mathbf{X})$. This is an unrealistic constraint, but can be accounted for by further elaboration of the model.

A suitable diffusion model may be one in which controlled substance is transferred from one note to it's neighbors in proportion to the concentration of controlled substance upon that note as illustrated in the following figure:



If this is the case, and β_0 is the diffusion constant, then the model can be expressed as:

$$y_1 = (1-\beta_0)x_1 + \beta_0 x_2$$
$$\vdots$$
$$y_i = (1-2\beta_0)x_i + \beta_0 x_{i-1} + \beta_0 x_{i+1}$$
$$\vdots$$
$$y_n = (1-\beta_0)x_n + \beta_0 x_{n-1}$$

Treating the diffusion model as an array of simultaneous equations one can define an invertible tri-diagonal matrix Γ_1 such that:

$$\Gamma_1 = \begin{pmatrix} 1-\beta_0, & \beta_0, & 0, & 0, & 0, & \ldots \\ \beta_0, & 1-2\beta_0, & \beta_0, & 0, & 0, & \ldots \\ 0, & \beta_0, & 1-2\beta_0, & \beta_0, & 0, & \ldots \\ \vdots & \vdots & \vdots & \vdots & \vdots & \ddots \end{pmatrix}$$

If $\hat{\mathbf{X}}$ are reconstructed values then:

$$\hat{\mathbf{X}} = \Gamma_1^{-1}\,\mathbf{Y}$$

Further work needs to be undertaken to estimate suitable values for β_0. Current methods are iterative and focus upon the effect that selection of various values for β_0 have upon the partial autocorrelation table.

Sampling considerations

The construction of suitable samples for Y_1 and Y_2 has to date tended to be of a "convenience" form and constitutes a grave weakness for this source of evidence. For a more systematic approach to construction of suitable samples:

1. Y_1, the sample of observations of banknotes known to be associated with the supply of controlled substances, is highly constrained as there are limited numbers of "known associated" cases. Do we:
   (a) Sample banknotes from those found to be guilty of controlled substance supply?
   (b) Sample banknotes from those who have confessed to controlled substance supply?

   Or have some other criteria for deciding which banknotes belong in this category.
2. Y_2, the sample of observations from banknotes from general circulation, should be randomly selected from all sources of banknotes not associated with the supply of controlled substances. In any specific case it may:
   (a) Have to be conditioned upon the geographical location from which the suspect banknotes, Y_3, were seized.
   (b) Have to be conditioned upon the specific temporal period.
   (c) Have to be conditioned on other factors such as banknotes from builders, and other groups who tend to frequently deal with paper money.

The sample Y_1 may also have to be geographically and temporally conditioned.

Concluding comments

It should be noted that this sort of evidence is not in any sense direct evidence in the same way that a DNA profile, or fingerprint might be, but is a form of "indirect" evidence, the epistemological status of which is currently under investigation.

Acknowledgments

The author should like to acknowledge Lawrence Dijourdy and Franco Taroni of ISPC, Lausanne, Switzerland.