The fallacy in the attempted proof of the Riemann hypothesis by Liu, Rybachuk and Liu

Note by G.J.O. Jameson

The fallacy is formula (3.7). Correctly, the integration here is on $(-\infty, 0]$, not $[0, \infty)$. This is easily seen by substituting $y = -z$ in (3.6). Equally, it results by applying (3.4) with $t < 0$, substituting $y = \frac{1}{2}t \ln x$, then writing $t = -u$. Actually, (3.4) says the same when applied to $-t$, because both sides are odd functions of t. No new information can be expected from writing it for $-t$.

There is no basis for (3.7) as stated, and in fact this integral will be divergent unless $\sigma = \frac{1}{2}$, because $\omega(e^{-2y/t}) \sim \frac{1}{2}e^{y/t}$ as $y \to \infty$.

Reference

Xiang Liu, Rybachuk Ekaterina and Fasheng Liu, A direct proof for Riemann hypothesis based on Jacobi functional equation and Schwarz reflection principle, *Advances in Pure Math.* 2016, 6, 193–200.